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Rotating bound states of dissipative solitons in systems of reaction-diffusion type

A.W. Liehr[1,a], A.S. Moskalenko[1], Yu.A. Astrov[1,2], M. Bode[1,3], and H.-G. Purwins[1]

[1] Institut für Angewandte Physik, Corrensstr. 2/4, 48149 Münster, Germany
[2] A.F. Ioffe Physico-Technical Institute, Russian Academy of Sciences, 194021 St. Petersburg, Russia
[3] Fraunhofer-Institut für Autonome Intelligente Systeme AIS, Schloss Birlinghoven, 53754 Sankt Augustin, Germany

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Abstract. We report on the formation of stable rotating bound states consisting of self-organized well localized solitary structures with particle-like behaviour in systems of reaction-diffusion type. These dissipative solitons are detected in an experimental planar d.c. gas-discharge system with a high ohmic barrier, as well as in numerical solutions of related three-component reaction-diffusion equations where the formation of rotating bound states is investigated in the context of a particle ansatz.

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Well localized self-organized solitary structures with particle-like behaviour are commonly observed in nature. In the case of dissipative systems we refer to them as dissipative solitons (DS), following references [1,2]. They exist in biological systems as nerve pulses [3], in chemical systems as concentration drops of chemical reagents [4–6], in optical systems as intensity bulbs [7,8], and as current filaments in semiconductor devices [9–11], or gas-discharge systems [1,12–14]. The modelling of these nonlinear dissipative systems often leads to reaction-diffusion equations [11,15–19] (in case of optical systems with cross-diffusion terms [7,8]). Recent investigations have shown that DSs are generic particle-like structures that can interact with each other. Typical interaction phenomena observed in experimental and theoretical systems are scattering, formation of molecule-like bound states, generation, or annihilation [6,12,13,16,20–25].

In this article we report on the experimental observation of rotating bound states of dissipative solitons in a planar d.c. driven gas-discharge system with a high ohmic barrier. In order to understand the results phenomenologically from the point of view of pattern formation, we investigate the formation of rotating bound states in a related three-component reaction-diffusion system, which has recently been incorporated in the detection of the drift-bifurcation of dissipative solitons in the pattern forming gas-discharge system [14,26].

[a] e-mail: obi@uni-muenster.de
http://www.uni-muenster.de/Physik/AP/purwins/struktur

The experimental system [27] is a variant of the electronic device initially designed for the high speed conversion of infrared images to the visible [28]. Essentially, it consists of two planar electrodes, one of which is a semiconductor wafer made of silicon doped with zinc. The other electrode consists of ITO (indium tin oxide) deposited on a glass substrate, which is transparent with respect to visible light (for parameters of the system see Fig. 1). In order to provide a high Ohmic cathode the semiconductor is cooled. The current of the system can be easily controlled by varying the d.c. feeding voltage U_0, or alternatively, adjusting the specific conductivity of the semiconductor wafer via the internal photoeffect by proper illumination of the wafer. If the value of U_0 is high enough, a self-sustained gas discharge is ignited. It turns out that the luminance emitted from the discharge space is proportional to the current density in the gap [27]. In this way, the information about the current density can be obtained by recording the luminance using commercial CCD cameras.

For proper parameters of the experimental system, stationary clusters of DSs [19], or patterns composed of a small number of moving DSs [12], may be generated in the device. In the present study, we concentrate on the investigation of bound states of two DSs which rotate around the center of the cluster. An example of this dynamical pattern is represented in Figure 1. This pattern is generated by taking advantage of the self-completion scenario described in [19], for example. Thereby, starting from a state with one DS, the global current is increased until

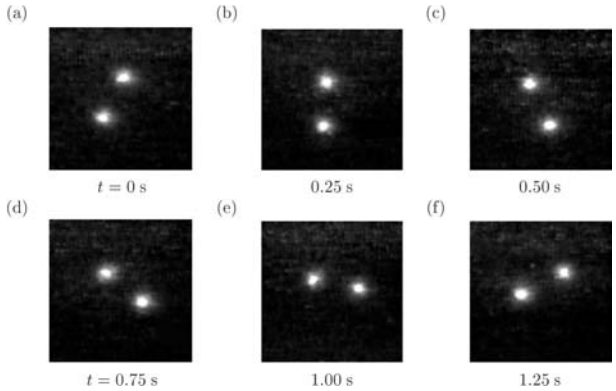

Fig. 1. Luminance distribution from the discharge gap of the gas-discharge system. The whole series represents a rotation of approximately 180°. The parameters are the following, gas: N_2, gas pressure: 2.0×10^4 Pa, semiconductor: Si⟨Zn⟩, semiconductor temperature: 90 K, thickness of gas layer: 0.8 mm, thickness of semiconductor wafer: 1.0 mm, specific conductivity of wafer 4.5×10^{-8} $\Omega^{-1}\text{cm}^{-1}$, overall supply voltage: 1.9 kV, resistor included into the circuit in series: 5.0×10^6 Ω, diameter of the active gas space: 20 mm, additional characteristic of the measurement: global current: 3.2 μA, exposure time for a frame is less than 1 msec, size of the presented domain: 9×9 mm^2.

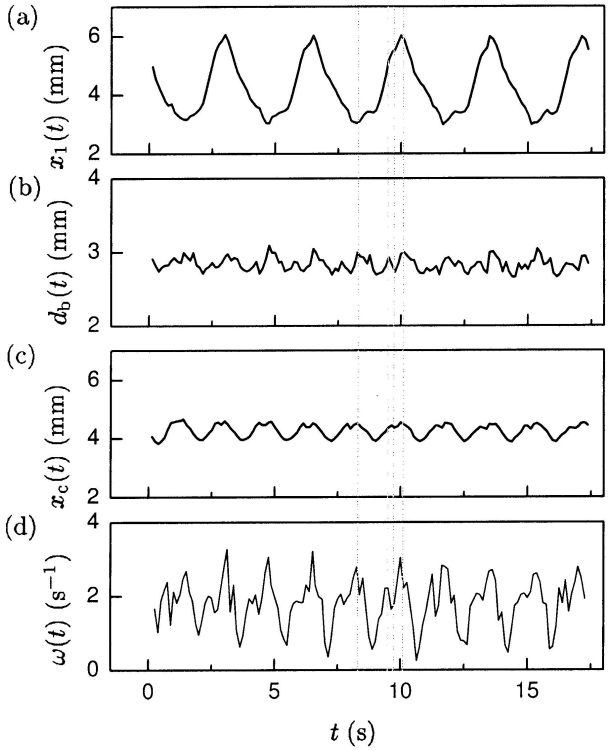

Fig. 2. Dynamical behaviour of the rotating pair of DSs shown in Figure 1. Gray vertical lines clarify the connection between some discontinuous changes of the slopes of curves shown in parts (a)-(d). (a) Projection of the position of an individual DS on the x-axis. (b) Distance $d_b(t)$ between the two DSs of the bound state. (c) Projection of the center of the bound state on the x-axis. (d) Angular velocity $\omega(t)$ of the bound state.

a second DS is ignited in the neighbourhood of the initial one. In the case of Figure 1 this pair of DSs, or this "molecule", as we may call it, is a stable pattern which rotates counterclockwise. We remark that a further increase of the global current leads to the appearance of additional DSs, whereas a more complicated dynamical behaviour can be observed as a consequence of the interaction between DSs.

In Figure 2a we plot the x-coordinate of the upper right DS of Figure 1a, which is $x_1(t)$, as a function of time for five periods of the rotation, thereby revealing a periodic behaviour. In each period the same characteristic deviation from sinusoidal motion is visible, indicating the influence of spatial inhomogeneities. The time dependence of the distance $d_b(t)$ between the centers of the individual DSs is depicted in Figure 2b. We observe that $d_b(t)$ varies periodically with half the period of $x_1(t)$. This correlation is due to the fact that both DSs are identical and are therefore identically influenced by both spatial inhomogeneities and boundaries. In Figure 2c the x-coordinate of the center of the molecule, defined as $x_c(t)$, is shown. It reveals a periodic dependence on time with characteristic fine structure which is correlated to the fine structure of Figure 2a. Furthermore, we observe that the periodic motion of the center is accompanied by strong changes in the angular velocity $\omega(t)$ (Fig. 2d).

In summary, we may state that the experimental results prove the existence of stable rotating pairs of DSs. Such a behaviour is different from that revealed in [12,29], where bound states of DSs have been observed in transient. The angular velocity is not constant, an effect that we attribute to the influence of spatial inhomogeneities in the device, presumably, in the semiconductor wafer. Due to the fact that the distance between the DSs within the cluster is relatively small with respect to the diameter of the active gas space, we assume that the boundary does not effect the dynamical behaviour of the cluster. This is in contrast to rotating periodic patterns whose dynamics is determined by the geometry and the boundaries of the domain [30–35].

To interpret the occurrence of rotating bound states of DSs in the experimental device, we recall that for such systems a phenomenological qualitative activator-inhibitor reaction–diffusion model has been proposed [36]. In this model the activating component is related to the avalanche multiplication of charged carriers in the gap, while the voltage drop at the semiconductor plate plays the role of the inhibitor. Two-component reaction-diffusion models allow for the qualitative understanding of many stationary patterns in planar d.c. gas-discharge systems, and other extended dissipative systems, e.g. Turing structures [18,27,37,38], stationary DSs [1,17,39,40] and their bound states [18,41]. Such models were also used to investigate moving DSs in more than one spatial dimension [42,43], whereas a single moving DS could be stabilized by a global feedback term. However, as it

was discussed in references [16,44], the global feedback term cannot suppress the growth of antisymmetric combinations of the unstable modes of two distinct DSs [45]. Another work [46] shows analytically that a moving two-dimensional DS exists, in principle, in a delicate limiting case of the standard FitzHugh-Nagumo model. Nevertheless, this result has not been confirmed by numerical simulations. The difficulties of this approach can be easily overcome by introducing a second inhibiting component in a phenomenological manner, so that the description of multiple stable moving DSs becomes possible [16]. In the context of planar d.c. gas-discharge systems, such a component can be related to the voltage drop at a gas region close to one of the electrodes [47]. Therefore, in the present analysis we apply a three-component reaction-diffusion system with one activator and two inhibitors [44]:

$$u_t = D_u \Delta u + \lambda u - u^3 - \kappa_3 v - \kappa_4 w + \kappa_1, \tag{1}$$

$$\tau v_t = D_v \Delta v + u - v, \tag{2}$$

$$\theta w_t = D_w \Delta w + u - w, \tag{3}$$

$$u = u(\vec{r},t),\ v = v(\vec{r},t),\ w = w(\vec{r},t),\ \vec{r} \in \Omega \subset \mathbb{R}^2,$$

$$D_u,\ D_v,\ D_w,\ \tau,\ \theta,\ \lambda,\ \kappa_3,\ \kappa_4 \geq 0.$$

Investigations of equations (1–3) in the parameter limit $D_v \to 0$ and $\theta \to 0$ have shown that stable stationary DS solutions undergo a supercritical bifurcation to moving DSs at $\tau_c = \frac{1}{\kappa_3}$ [48], thereby loosing their spherical symmetry, which is an analog to spontaneous parity breaking [49,50]. The same holds for bound states of dissipative solitons, which can undergo a transition to rotation or translation at τ_c [16]. Note, however, that the situation is more difficult for $\theta > 0$. For this case, and that concerning a bound state of two dissipative solitons it has been shown that the bifurcation point of the rotational bifurcation of the bound state is smaller than the drift bifurcation point of the bound state, which in turn is smaller than the drift bifurcation point of the unbound dissipative soliton [51, 52].

Here we focus on the parameter limit $D_v \to 0$ and $\theta \to 0$, for which unbounded DSs for $\tau > \tau_c$ move with an intrinsic velocity given by:

$$c_0(\tau) = \kappa_3^{\frac{3}{2}} \sqrt{\frac{\tau - \tau_c}{Q}} \tag{4}$$

[14], where Q is a scalar shape factor obtained from the radial activator distribution $\bar{u}(r)$ of a stationary circular symmetric DS solution as such:

$$Q = \frac{3}{4} \frac{\int_0^\infty \mathrm{d}r\ \left(\bar{u}_{rr}^2 r + \bar{u}_r^2 / r\right)}{\int_0^\infty \mathrm{d}r\ \bar{u}_r^2 r}. \tag{5}$$

Near the onset of propagation, $\tau = \tau_c$, the dynamics of DSs can be reduced by means of a perturbation approach to a set of ordinary differential equations for the positions of the DSs $\vec{p}_i(t)$, and the vectors $\vec{\alpha}_i(t)$ which represent the shift between the centers of activator distribution u and centers of slow inhibitor distribution v for each single DS [16]. We note that $\vec{\alpha}_i(t) \neq 0$ is the origin of the motion of DSs, and is referred to as the amplitude of the propagator mode. The dynamics for two interacting DSs are described by

$$\dot{\vec{p}}_1 = \kappa_3 \vec{\alpha}_1 - F(d)\,(\vec{p}_2 - \vec{p}_1)\,, \tag{6}$$

$$\dot{\vec{\alpha}}_1 = \kappa_3^2 \left(\tau - \frac{1}{\kappa_3}\right) \vec{\alpha}_1 - \kappa_3 Q\,|\vec{\alpha}_1|^2\,\vec{\alpha}_1 - F(d)\,(\vec{p}_2 - \vec{p}_1)\,, \tag{7}$$

$$\vec{p}_1, \vec{\alpha}_1 \in \mathbb{R}^2,$$

with an equivalent set of equations for the second DS. The scalar interaction function $F(d)$ depends on the distance $d = |\vec{p}_2 - \vec{p}_1|$ between the DSs and, like the scalar shape factor Q, can be easily computed from a stationary DS solution [16]. That is to say, the complicated dynamics of multiple moving DSs can be described and understood using the dynamics of individual interacting particles with an intrinsic velocity. Comparison between the dynamics of DSs computed from the field equations (1–3) and that obtained from the reduced dynamics (6–7) near the bifurcation point ($\tau \gtrsim \tau_c$), show good agreement [16,53]. Note, that for $\tau \leq \tau_c$ the DSs are stationary and can only start to move from mutual interactions [41]. For large distances d, the interaction function $F(d)$ vanishes, and for $\tau > \tau_c$ the DSs move with their intrinsic velocity (4). In this context, we note that we can vary the intrinsic velocity $c_0(\tau)$ as a function of relaxation time τ of the slow inhibitor v.

Solutions to equations (1–3) are computed numerically using a Crank-Nicholson time stepping scheme with constant discretization in space and time and a red-black Gauß-Seidel iteration with successive over-relaxation. The equation of the fast inhibitor w, equation (3), is treated with a multigrid solver in the parameter limit $\theta = 0$ [23].

In order to investigate the formation of the rotating bound states of the DSs, we have chosen parameters which provide DS solutions with tails which decay in an oscillatory manner towards the homogeneous background state. This leads to an interaction function which also shows oscillatory behaviour with attractive ($F(d) < 0$) and repulsive ($F(d) > 0$) regions of interaction [16,41]. For the parameters given in the caption of Figure 3, the interaction function $F(d)$ can be approximated by the following fitting function

$$F(d) = -\frac{6.87 \times 10^{-4}}{d^{3/2}} e^{-15.7\,d} \cos(43.15(\,d - 0.199)), \tag{8}$$

which is calculated from the activator distribution of a stationary DS solution [16,54]. The reduced description, equations (6–7), of the dynamics of the DSs provide a powerful tool for investigating the interaction of DSs in a fast and convenient way. This is because the infinite number of degrees of freedom of the field equations (1–3)

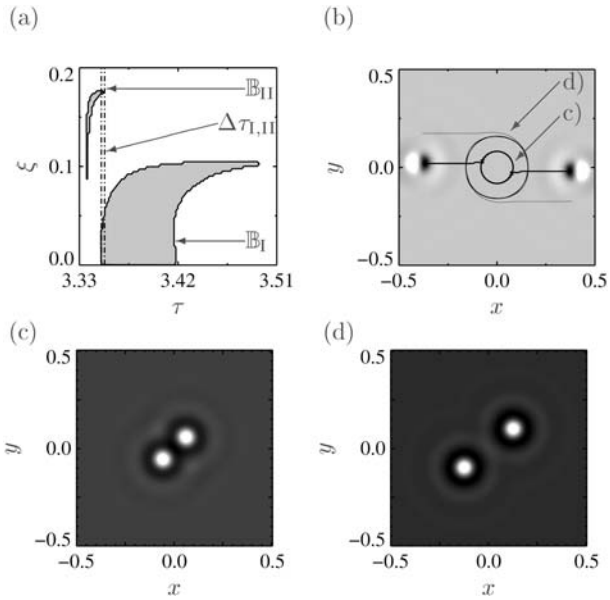

Fig. 3. Stable rotating bound states of DSs in a three-component reaction-diffusion system. (a) Solutions of the reduced dynamics (6–7) for varied time scale constant τ, varied impact parameter ξ and initial conditions $\vec{p}_{1,0} = (-0.4, \xi)$, $\vec{p}_{2,0} = (0.4, -\xi)$, $\vec{\alpha}_{1,0} = (c_0(\tau)/\kappa_3, 0)$, and $\vec{\alpha}_{2,0} = (-c_0(\tau)/\kappa_3, 0)$, computed with the interaction function (8) and using $Q = 1950$ [16]. (b) Difference of activator u and slow inhibitor v distributions corresponding to $(\tau, \xi) = (3.35, 0.02) \in \mathbb{B}_I$, which illustrates the initial shifts $\alpha_{1,0}$ and $\alpha_{2,0}$ between the centers of activator distribution u and slow inhibitor distribution v for the left and right DS. Dark (light) colors denote regions of $u > v$ ($u < v$), respectively. The image also shows two pairs of trajectories computed from solutions of the field equations (1–3), leading to the formation of two different counterclockwise rotating bound states, which are depicted in (c) and (d). (c) Activator distribution u of a counterclockwise rotating bound state computed from the initial conditions depicted in (b), with binding distance $d_I = 0.164$ and angular velocity $\omega_I = 5.44 \times 10^{-3}$. (d) Activator distribution u of a counterclockwise rotating bound state computed from $(\tau, \xi) = (3.35, 0.174) \in \mathbb{B}_{II}$, with binding distance $d_{II} = 0.320$ and angular velocity $\omega_{II} = 2.90 \times 10^{-3}$. Equations (1–3) are solved with parameters $D_u = 1.1 \times 10^{-4}$, $D_v = 0$, $D_w = 9.64 \times 10^{-4}$, $\lambda = 1.01$, $\kappa_1 = -0.1$, $\kappa_3 = 0.3$, $\kappa_4 = 1$, $\theta = 0$ on a domain $[-1, 1] \times [-1, 1]$ with no-flux boundary conditions, and a discretization of $\Delta x = 5 \times 10^{-3}$ and $\Delta t = 0.1$.

are reduced, in our case, to four degrees of freedom per soliton on a two-dimensional domain.

We now want to find out under which conditions the rotating pairs of DSs form. This may depend on the parameter τ, and in addition on the initial conditions, which are parameterized by an impact parameter ξ, such that two DSs start far away from each other ($d \geq 0.8$) with anti-parallel velocity vectors (see caption of Fig. 3). We now solve equations (6–7) for various time scale constant τ and impact parameter ξ in the two-dimensional parameter interval $\tau \times \xi = [\tau_c, 3.51] \times [0, 0.2]$, using a fifth order Runge-Kutta-Verner method. The results are shown in Figure 3a, where the shaded area denotes the formation of rotating pairs of DSs. We find a large area $\mathbb{B}_I$ and a smaller one $\mathbb{B}_{II}$. The area $\mathbb{B}_I$ does not reach the $\xi = 0$ axis. This limiting case leads to the formation of non-rotating bound states [16]. In order to visualize the difference between the states corresponding to regions $\mathbb{B}_I$ and $\mathbb{B}_{II}$, we have chosen one parameter pair (τ, ξ) from each of the parameter ranges $\mathbb{B}_I$ and $\mathbb{B}_{II}$, and solved the field equations (1–3) with corresponding initial conditions. The results of these simulations are shown in Figure 3b as trajectories of the center coordinates computed from the activator distribution u of the respective DS. The trajectories show the formation of two different rotating bound states each one consisting of two DSs with binding distances $d_I = 0.164$ and $d_{II} = 0.320$, which are slightly larger than the corresponding roots of the interaction function $F(d)$ at $d_{0,I} = 0.162$ and $d_{0,II} = 0.308$.

Momentary activator distributions for the resulting rotating bound states are shown in Figures 3c and 3d. The counterclockwise rotating bound state of Figure 3c is result of the initial conditions visualized in Figure 3b corresponding to $(\tau, \xi) \in \mathbb{B}_I$. This bound state can only form if the DSs are fast enough to overcome the repulsive interaction for $d \in [0.235, 0.308]$, with $F(d) > 0$. On the other hand, if they are too fast ($\tau > 3.494$), the attractive force is not strong enough to compensate the "centrifugal forces", and rotating bound states cannot exist. We should note, that for many particle interactions of fast moving DSs ($\tau \gtrsim 3.41$), additional phenomena such as generation and annihilation of DSs are observed, which are, however, beyond the scope of the reduced dynamics approach [23].

Because of the exponentially decaying interaction function $F(d)$, the attractive interaction between DSs at a binding distance d_{II} is much weaker than it is at d_I. Therefore, the second bound state can only form for relatively small velocities, i.e. for $\tau < 3.353$ in our case. Figure 3d shows an example of this bound state computed by solving the field equations (1–3) numerically, using initial conditions $(\tau, \xi) \in \mathbb{B}_{II}$. In this case, the DSs are fast enough to overcome the repulsive region $d \in [0.308, 0.380]$, but are slow enough to be captured by the attractive interaction at a binding distance d_{II}. Within a small interval $\tau \in \Delta\tau_{I,II} = [3.350, 3.352]$, both rotational bound states can be reached. This interval clearly denotes bistability of the system in the sense that two independent DSs may form two different rotating states in the course of their interaction. This result is proved by solving the field equations (1–3), presented in Figures 3c–3d, which have been computed using $\tau = 3.35$ and values of ξ according to $\mathbb{B}_I$ and $\mathbb{B}_{II}$.

We have presented experimental and numerical investigations of the rotating bound states of two DSs. These bound states consist of individual DSs, which, to a large extent retain the free soliton properties. Such localized self-organized solitary dissipative patterns are characterized by well defined binding distances of the DSs, and, in principle, rotate with constant angular velocity. The

deviation from constant angular velocity observed in the experiment is attributed to spatial inhomogeneities. For the numerical investigations we have used two approaches: the first based on the field equations (1–3), and the second where these equations are reduced to a system of ordinary differential equations with a small number of components. The numerical results from both approaches are in good agreement. Recently, rotating bound states of two localized states were also reported in the vector complex Ginzburg-Landau equation [55], where a single localized state is stationary in the absence of the other one. In contrast, the DSs investigated in the present work move with an intrinsic velocity, which is crucial for the formation of the rotating bound states. Finally, we want to remark that the formation of rotating clusters of more than two DSs can be easily understood on the basis of the results presented here.

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